NOMAN SADAQAT

COMSATS University Islamabad
Department of Physics
Final Examination Fall 2025
Subject: Probability Theory and Random variable

Program: ELC
Semester: 4th
Date: 5th January 2026.

Total Marks: 50
Time Allowed: 180 Minutes
Instructor: Ms. Mudassrah

Instructions:

- **Attempt all questions.**
- **All questions carry equal marks. Normal distribution table and calculators are allowed but sharing is strictly prohibited.**

Question No. 01: A study was made on the amount of converted sugar in a certain process at various temperatures. The data were coded and recorded as follows:

Temperature(x)	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Converted sugar(y)	8.1	7.8	8.5	9.8	9.5	8.9	8.6	10.2	9.2	9.3	10.5

a. Estimate the linear regression line.
b. Estimate the mean amount of converted sugar produced when the coded temperature is 1.75.

Question No. 02: The joint density for the random variables *(X, Y)*, where *X* is the unit temperature change and *Y* is the proportion of spectrum shift that a certain atomic particle produces, is

$$f(x,y) = \{10xy^2, \quad 0 < x < y < 1$$
$$= 0 \quad \text{elsewhere}$$

a. Find the marginal densities *g(x), h(y),* and the conditional density *f(y | x).*
b. Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased by 0.25 unit.

Question No. 03:

Let X be a random variable with the following probability distribution:

X=x	-2	3	5
f(x)	0.3	0.2	0.5

Find the standard deviation of X.

Question No. 04:

Suppose that for a very large shipment of integrated-circuit chips, the probability of failure for any one chip is 0.10. Assuming that the assumptions underlying the binomial distributions are met, find the probability that at most 3 chips fail in a random sample of 20.

Question No. 05:

Imperfections in computer circuit boards and computer chips lend themselves to statistical treatment. For a particular type of board, the probability of a diode failure is 0.03 and the board contains 200 diodes.

a. What is the mean number of failures among the diodes? What is the variance?
b. The board will work if there are no defective diodes. What is the probability that a board will work?

Question No. 06:

For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

Question No. 07:

An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Question No. 08:

In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, how many possible selections are there?

Question No. 09:

The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?

Question No. 10:

A continuous random variable X that can assume values between $x=2$ and $x=5$ has a density function given by $f(x) = 2(1+x)/27$

a. Find $P(X < 4)$
b. Find $P(3 \leq X < 4)$

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