COMSATS UNIVERSITY ISLAMABAD (CUI)

DEPARTMENT OF COMPUTER SCIENCE

FINAL TERM EXAMINATION FALL-2025
BS (CS)--II SEMESTER
BS (SE)--II SEMESTER
BS (DS/AI/CYS)– II SEMESTER

Course:CSC102-Discrete Structure**Dated:**
Maximum Marks: 50 **Time Allowed:**180 Minutes

- *All questions are self-explanatory and require no further explanations during exam time.*
- *Make sure that you have signed the attendance sheet before leaving the examination room.*
- *Return the question paper along with the answer sheet.*
- *Attempt all questions.*

CLO – 1: Apply symbolic propositional and predicate logic to determine the most effective solutions of a given problem:

Q.# 1. **[Marks:3+3]**

1.1. Let
s = "The router can send packets to the edge system";
a = "The router supports the new address space";
r = "The latest software release is installed."

Determine the "*truth value*" of above propositions so that the following system specifications must be consistent(i.e., all given statements form must be true simultaneously).

"The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it is necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed, The router does not support the new address space."

1.2. Write the following statement in English, using the predicate S(x, y) for "x shops in y", where x represents people and y represents stores:

$$\exists x_1 \exists y \forall x_2 [S(x_1, y) \wedge (x_1 \neq x_2 \rightarrow \sim S(x_2, y))]$$

CLO – 2: Apply formal logic proofs and reasoning to construct a sound argument

Q.# 2. **[Marks: 2.5+2.5+2+3 = 10]**

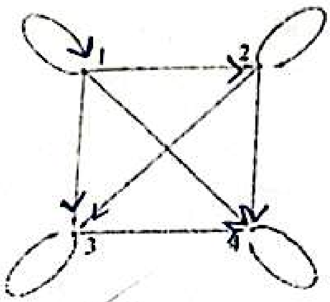
2.1 Prove by contraction that there exist no integers a and b for which 21a + 30b = 1.
2.2 Prove given statement directly: Let a and b are integers such that $2a = b^2 + 3$.Prove that a can be expressed as the sum of three squares.
2.3 Give a counter example to disprove the given statement: "*The square of every real number is positive*".
2.4 Prove by mathematical induction for all integers n≥1

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1$$

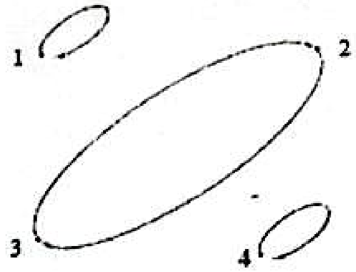
(CLO – 3: Solve a computing problem using a specific set, function, or relation model.)

Q. # 3. [Marks:5+5].

3.1 Perform set operations "***Union***" and "***Intersection***" on the "***relations***" R_1 and R_2 defined by the following *directed graphs*. Represent your results in *matrix* form.



R₁



R₂

3.2 Consider the following two-piece wise functions.

$$f(x)=\begin{cases}2x+1, & \text{if } x\le 0\\ x^2, & \text{if } x>0\end{cases}\quad g(x)=\begin{cases}-x, & \text{if } x<2\\ 5, & \text{if } x\ge 2\end{cases}$$

Find *gof*, and then find its inverse.

(CLO – 4: Use recurrence relation and counting formalisms to solve real-world problems.)

Q. # 4. [Marks:5+5].

4.1 A small company tracks the number of **new customer inquiries** it receives each day.The number of inquiries on day $n$is described by the following recurrence:

$$I_n=\begin{cases}2, & n=1\\ I_{n-1}+3, & \text{for } n>1\end{cases}$$

The analytics team wants to evaluate how inquiry growth on earlier days compares with later days.They define the following expression:

$$\sum_{i=1}^{2}\sum_{j=i+1}^{4}\left(I_i^2-I_j+1\right)$$

Compute the value of this double summation using the values of the recurrence.

4.2 A palindromic integer or palindrome is a positive integer whose decimal expansion is symmetric and that is not divisible by 10. In other words, it is an integer that reads the same backward as forward. For example, the following integers are all palindromes: 1,8,11,99,101,131,999,1234321

a) How many five digit palindromes are there?

b) How many are odd? How many are even?

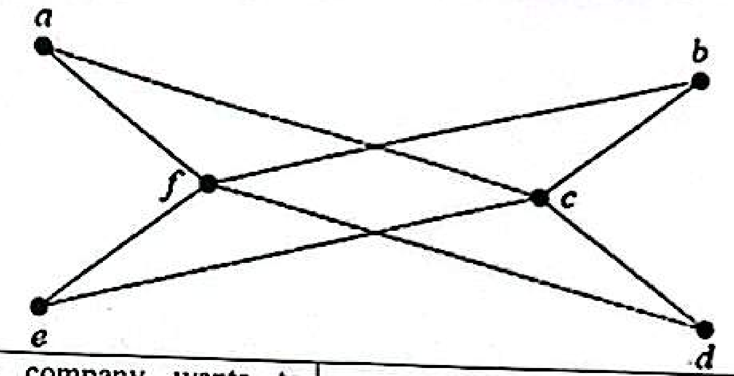
9 10 10

Q. # 5. [Marks:2+3+3+6=14].

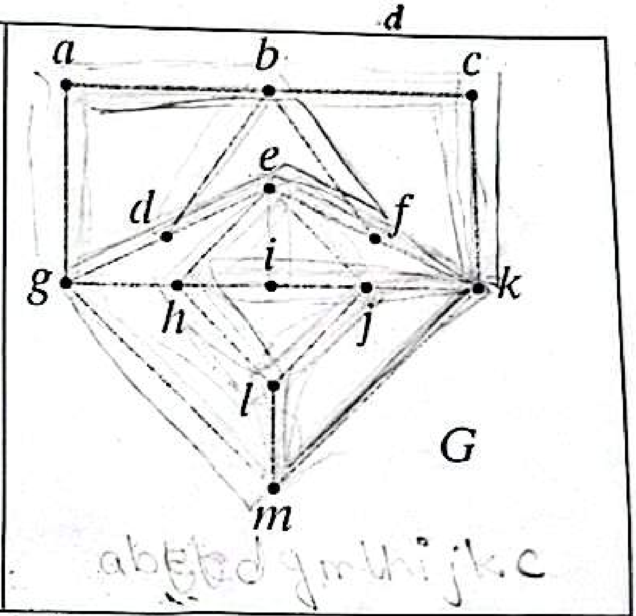
5.1 Draw a graph having the given properties orexplain why no such graph exists.

Simple graph; six vertices having degrees 1, 2, 3, 4, 5, 5

5.2 Determine whether the following graph is bipartite or not. If so, identify the vertex sets V_1 and V_2

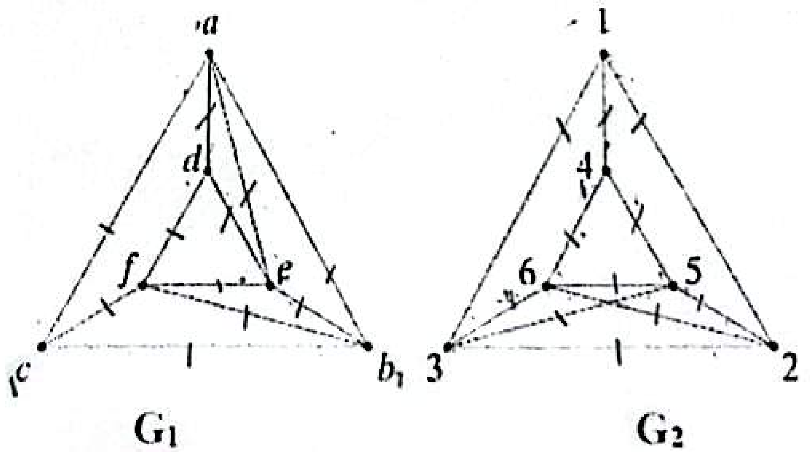


5.3 A courier company wants to determine whether it is possible for a driver to start from the warehouse, visit **each delivery zone exactly once**, and return to the warehouse without revisiting any zone. The delivery zones and roads between them are represented using an undirected graph. Given the graph, **determine** given graph contains a "***Hamiltonian cycle***". If there is a Hamiltonian cycle,exhibit it; otherwise, givean argument that shows there is no Hamiltonian cycle.



5.4 A social media platform models friendships using an **undirected simple graph.** Two different teams built visualizations of the same friendship data but used different layouts.You are given the following two graphs (Graph **G1** and Graph **G2**), each representing friendships between **six users** where vertices represent users and edges represent mutual friendships. Determine whether **Graph G1** and **Graph G2** represent the *same* friendship network (i.e., whether they are **isomorphic**).If the graphs are isomorphic, find functions *f* and *g*; otherwise give an invariant that both graphs do not have in common.

ac



End