



Department of Computer Engineering

FA23BCE088

CPE251 Probability Methods in Engineering

Terminal Examination Spring 2025

Instructor: Dr. Babar Mansoor

Max Marks: 50

Date: July 09, 2025.

Class: BCE4A/B

Time Allowed: 3 Hours

Instructions:

- The question paper consists of seven (07) questions and is printed on both sides of the page. You are required to attempt all questions.
- Read the questions carefully before attempting.
- Write your name, roll number, course code, and section clearly on the answer sheet.
- Use of mobile phones, smartwatches, Bluetooth devices, or any type of smart electronic gadgets is strictly prohibited.
- Follow all verbal instructions given by the invigilators.
- Do not communicate with other students.
- Any form of cheating or misconduct will result in immediate disqualification and disciplinary action.
- Write legibly on the answer sheet.

Q(1) (5 Marks) (CLO1, C3) A student has a 70% chance of preparing for a test. If the student prepares, the probability of passing is 0.9. If the student does not prepare, the probability of passing is 0.3. Given that the student passed the test, compute the probability he had prepared for it.

Q(2) (5 Marks) (CLO2, C3) Let X be a discrete random variable (RV) with the following probability mass function (PMF): $p_X(1) = 0.1$, $p_X(2) = 0.3$, $p_X(3) = 0.4$, and $p_X(4) = 0.2$. Compute and sketch the cumulative distribution function (CDF), $F_X(x)$, of X .

Q(3) (5 Marks) (CLO3, C3) Let X be a continuous RV with the probability density function (PDF) given by:

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- Find the conditional PDF of X , given that $X > 0.5$.
- Sketch both the original and the conditional PDF.
- Compute $P(0.6 \leq X \leq 0.9 | X > 0.5)$.
- Compute $E[X | X > 0.5]$ and $VAR[X | X > 0.5]$.

Q(4) (5 Marks) (CLO3, C3) Let X be an exponential RV with parameter $\lambda = 2$, i.e.,

$$f_X(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Define a new random variable $Y = \sqrt{X}$.

- (a) Compute the PDF of Y using CDF method. Clearly indicate the support of Y .
- (b) Compute the probability $P(1 \leq Y \leq 2)$.

Q(5) (10 Marks) [CLO4, C4] Consider the joint PMF of two discrete RVs X and Y defined as:

$$P_{X,Y}(x,y) = \begin{cases} \frac{1}{9}, & x \in \{1, 2, 3\}, y \in \{1, 2, 3\} \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find the marginal PMFs $p_X(x)$ and $p_Y(y)$.
- (b) Find the probability $P(X + Y \leq 4)$.
- (c) Are X and Y independent or not? Justify your answer.

Q(6) (10 Marks) [CLO4, C4] Let the joint PDF of two continuous RVs X and Y be given by:

$$f_{X,Y}(x,y) = \begin{cases} 6(1-y), & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Sketch the support of $f_{X,Y}(x,y)$.
- (b) Identify and shade the region corresponding to $\{(x,y) : x + y \leq 1\}$, then compute the probability.
- (c) Find and sketch the marginal PDFs $f_X(x)$ and $f_Y(y)$.
- (d) Find whether the X and Y are independent or not. Justify your answer.
- (e) Find the covariance of X and Y .

Q(7) (10 Marks) [CLO4, C4] Let the joint PDF of two continuous RVs X and Y be given by:

$$f_{X,Y}(x,y) = \begin{cases} \frac{4}{\pi}, & x^2 + y^2 \leq 1, x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Sketch the support of $f_{X,Y}(x,y)$.
- (b) Verify that $f_{X,Y}(x,y)$ is a valid PDF.
- (c) Compute the probability $P(X^2 + Y^2 \leq 1/4)$.
- (d) Find the joint CDF $F_{X,Y}(x,y)$.

———— All the Best! ————