COMSATS UNIVERSITY ISLAMABAD (CUI)

DEPARTMENT OF COMPUTER SCIENCE

FINAL TERM EXAMINATION SPRING - 2026
BS (CS)— -II SEMESTER
BS (SE)— -II SEMESTER
BS (DS/AI/CYS)— II SEMESTER

Course: CSC102-Discrete Structure **Dated:** June 6, 2026
Maximum Marks: 50 **Time Allowed:** 180 Minutes

- *All questions are **self-explanatory** and require no further explanations during exam time.*
- *Make sure that you have **signed the attendance sheet** before leaving the examination room.*
- ***Return** the question paper along with the answer sheet.*
- *Attempt **all** questions.*

CLO – 1: Apply symbolic propositional and predicate logic to determine the most effective solutions of a given problem.

Q.# 1. **[Marks:2*3=6]**

1.1. Find the negation of the following statement.
"If you pay your membership dues, then if you come to the club, you can enter free."

1.2. Write the negation of the following predicate.

$$\forall x(\exists y \forall z P(x,y,z) \wedge \exists z \forall y P(x,y,z))$$

1.3. Let $P(x,y)$ denote "x is a factor of y" where $x \in \{1,2,3,\cdots\}$ and $y \in \{2,3,4,\cdots\}$. Let Q(y) denote "$\forall x[P(x,y) \rightarrow ((x=y) \vee (x=1))]$". When is Q(y) true?

CLO – 2: Apply formal logic proofs and reasoning to construct a sound argument

Q.# 2. **[Marks:3+3+4 = 10]**

2.1 Give a proof by contradiction to the following statement.

There is no rational number r for which $r^3 + r + 1 = 0$

2.2 Let $a_1, a_2, a_3, \ldots$ be (recursively) defined as follow:

$$a_n = \begin{cases} 3 & if\ n = 1 \\ 4 & if\ n = 2 \\ a_{n-2} + 2a_{n-1} & otherwise \end{cases}$$

Prove by mathematical induction that for every k > 0, a_{2k} is even and a_{2k-1} is odd.

2.3 Consider the following argument:

Some AI students learn Python.
Everyone who learns Python can build ML models.
∴ Some AI students can build ML models.

Translate above argument into symbolic form and then determine whether the argument is **valid or invalid**. Justify your answer using rules of inference.

(CLO – 3: Solve a computing problem using a specific set, function, or relation model.)

Q. # 3. **[Marks:3+3.5+3.5=10]**

3.1 A university is analyzing student enrollment patterns across different academic activities:

- Some students are enrolled in the **Data Science course.**
- Some students are enrolled in the **Artificial Intelligence (AI) course.**
- Some students are participating in **Research Projects.**

The administration makes the following claim:

$-3(3-2)$
$-3(6)$

"*The group of students who are enrolled in Data Science or AI, but excluding those AI students who are also involved in research, is exactly the same as the group of students who are either taking AI but not involved in research, or taking Data Science but not AI.*"

i. Translate the above statement into a **set-theoretic expression** using appropriate sets.

ii. Determine whether the claim is **true or false.** Justify your answer by constructing a **membership table (or Venn diagram).**

3.2 A **university's smart campus system** tracks how students interact with different facilities. Consider the set of locations: $A = \{1,2,3,4\}$, where:

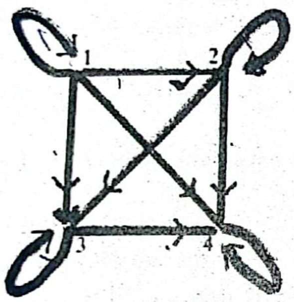
1 = Library   2 = Lab   3 = Cafeteria   4 = Hostel

Two relations are defined based on system logs:

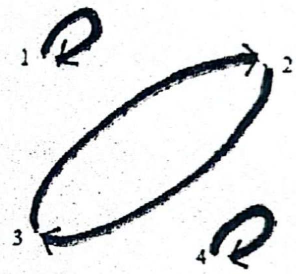
Relation R_1: "Direct movement" — there is a directed edge from location a to b if students frequently move directly from a to b. (Includes self-loops where students stay in the same location.)

Relation R_2: "Recommended transitions" — there is a directed edge from a to b if the system recommends moving from a to b based on usage patterns.

These relations are represented by the directed graphs shown in the figure.



R1



R2

i. Find the matrix representation of $R = R_1 \circ R_2$ using **Boolean matrix multiplication**

ii. Determine whether the resulting relation R is an equivalence relation or not. If resulting relation is not equivalence, then add minimum number of order pairs in R to make it equivalence.

F(2) =

3.3 A **temperature calibration system** is used in an industrial IoT setup. The system converts raw sensor readings into calibrated temperature values using a linear function $f(x) = ax + b$, where x is the raw sensor input. During testing, the following observations were recorded:

- When the sensor input is $x = 1$, the calibrated output is $f(1) = 1$.
- A calibrated temperature of 5 corresponds to a raw sensor input of $x = 2$, i.e., $f^{-1}(5) = 2$.

i. Determine the values of constants a and b.

ii. Find the inverse function $f^{-1}(x)$. Use the inverse function to compute $f^{-1}(-\frac{51}{2})$

(CLO – 4: **Use recurrence relation and counting formalisms to solve real-world problems.)**

Q. # 4. **[Marks:3+3+4=10]**

4.1 A distributed system assigns 6 identical tasks to 4 servers. Any server can handle multiple tasks. In how many ways can the tasks be distributed?

4.2 A data scientist is building a training dataset by randomly selecting 5 images from a pool of 8 categories. Each selection is independent, and categories can repeat. How many possible samples of 5 images can be created?

4.3 A tech company is tracking the growth of users on a new digital platform.

- On the first day, the platform has **1 active user**.
- Each day, the number of users **doubles due to sharing and referrals**.
- In addition, the platform gains **extra users from marketing campaigns**, where the number of new users added on day n is $2^n - 1$.

Let a_n represent the total number of users on day n.

i. Formulate a recurrence relation to model the total number of users.

ii. Using this relation, compute the total number of users on **day 6**.

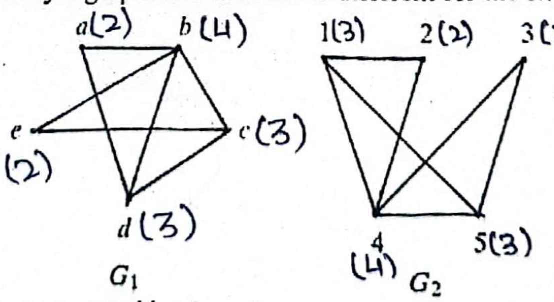
(CLO – 5: Solve real-world problems in computer science using appropriate forms of graphs and trees.

Q. # 5. **[Marks:2+4+3+5=14].**

5.1 Draw a graph having the given properties or explain why no such graph exists.

Simple graph; ~~six~~ Five vertices having degrees 2, 3, 3, 4, 4

5.2 Determine whether Graph G_1 and Graph G_2 are isomorphic. If the graphs are isomorphic, specify the bijective functions f and g between their vertex and edge sets. Otherwise, identify a graph invariant that is different for the two graphs.



G_1 G_2

5.3 Consider the graph represented by the adjacency matrix A. How many paths of length 4 exist from vertex d to vertex e. Find all these paths.

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}.$$

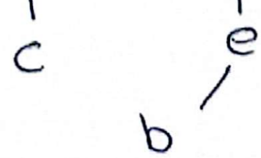
Circuit X ✓ ✓

Simple X X ✓

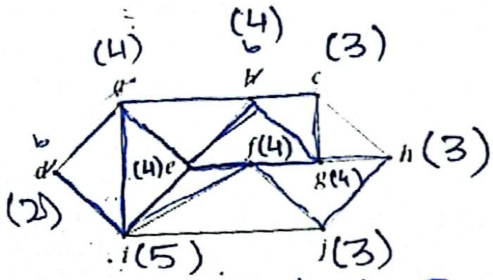
Path — edge repeat X

— Vertex repeat ✓

X



5.4 Consider the graph given on right side, and answer the following:



- i Write the "*adjacency matrix*".
- ii Determine whether the given graph is "*bipartite*" or not. If the graph is bipartite, specify the disjoint vertex sets A and B.
- iii Decide whether the graph has an "*Euler cycle*" (Circuit). If the graph has an Euler cycle, exhibit one; otherwise, give an argument that shows there is no Euler cycle.
- iv Determine whether or not the given graph contains a "*Hamiltonian cycle*". If there is a Hamiltonian cycle, exhibit it; otherwise, give an argument that shows there is no Hamiltonian cycle.
- v Verify that the number of vertices of odd degree in the graph is even.

abcghjFiеFgbe ida

Eular cycle — All vertex
All edge

Vertex may repeat

abcghjF eid a

End

A abcghjFi