COMSATS University Islamabad
Department of Computer Science
Mid Term Examination – Fall 2021

Class: (BS) CS/SE/AI/DS/CYS — Semester: 1st / 2nd
Course Code & Title: CSC102 Discrete Structures — Date: 20-11-2021
Time Allowed: 90 Minutes — Max Marks: 25
Instructors: Dr. Amir Hanif, Dr. Ahmed Kamran, Dr. Sheneela, Ms. Memoona

Name:___________________________ **Registration #**_________________

Instructions
1. Mobile phones are strictly prohibited in examination hall.
2. Use black or blue ball-pen only. Markers/lead pencils are not allowed.

	Q1	Q2	Q3	Max
Total	9	8	8	25
Obtained				

Question 1 CLO-1 Marks 9

1) Which of these are propositions? What are the truth values of those that are propositions? (3)

Statement	Is this a proposition?	Truth value [T/F] (in case of proposition)
a) Do not pass go.	Not a proposition	
b) What time is it?	Not a proposition	
c) There are no black flies in Maine.	Proposition	False
d) 4 + x = 5.	Not a proposition	
e) The moon is made of green cheese.	Proposition	False
f) 2n ≥ 100.	Not a proposition	

2) Let p, q, and r be the propositions **(1)**

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express the following propositions as an English sentence.

a) $p \rightarrow \neg q$ If I bought a lottery ticket this week, then I would not win the million dollar jackpot.
b) $p \vee (p \wedge \neg q)$ I bought a lottery ticket this week or I bought a lottery ticket this week and did not win the million dollar jackpot.

3) Let p, q, and r be the propositions **(1)**

p : Grizzly bears have been seen in the area.

q : Hiking is safe on the trail.

r : Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

a) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area. $r \rightarrow (q \leftrightarrow \neg p)$
b) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe $\neg q \wedge \neg p \wedge r$

4) Show that $\neg(p \rightarrow q) \rightarrow \neg q$ is a tautology using truth table. **(1)**

p	**q**	**¬q**	**p → q**	**¬(p → q)**	**¬(p → q) → ¬q**
T	**T**	**F**	**T**	**F**	**T**
T	**F**	**T**	**F**	**T**	**T**
F	**T**	**F**	**T**	**F**	**T**
F	**F**	**T**	**T**	**F**	**T**

5) Let P (x) be the statement “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++.”
Express the following sentences in terms of P (x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school. (3)

a) There is a student at your school who can speak Russian but who doesn’t know C++. $\exists x(P(x) \land \neg Q(x))$
b) Every student at your school either can speak Russian or knows C++. $\forall x(P(x) \lor Q(x))$
c) No student at your school can speak Russian or knows C++. $\neg\exists x(P(x) \lor Q(x))$

<u>Question 2 CLO-2 Marks 8</u>

1) For each of these arguments determine whether the argument is correct or incorrect and explain why. [4]

a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

(a) Let us assume:

$P(x) =$"x is a student in this class"

$Q(x) =$"x understands logic"

We can then rewrite the given statements using the above interpretations.

	Step	Reason
1.	$\forall x(P(x) \to Q(x))$	*Premise*
2.	$P(\text{Xavier})$	*Premise*
3.	$P(\text{Xavier}) \to Q(\text{Xavier})$	Universal instantiation from (1)
4.	$Q(\text{Xavier})$	Modus ponens from (2) and (3)

Step (4) means that "Wavier understands logic". This corresponds with the given conclusion and thus the argument is correct.

b) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

(b) Let us assume:

$P(x) =$"x is a computer science major"

$Q(x) =$"x takes discrete mathematics"

We can then rewrite the given statements using the above interpretations.

	Step	Reason
1.	$\forall x(P(x) \rightarrow Q(x))$	*Premise*
2.	Q(Natasha)	*Premise*
3.	P(Natasha) $\rightarrow Q$(Natasha)	Universal instantiation from (1)

There is no rule of inference that allows us to conclude P(Natasha), which means "Natasha is a computer science major", and thus the argument is incorrect.

2) For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. [4]

a) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."

(c) Let us assume:

$p =$"I am clever"

$q =$"I am lucky"

$r =$"I win the lottery"

We can then rewrite the given statements using the above interpretations.

	Step	Reason
1.	$p \vee q$	*Premise*
2.	$\neg q$	*Premise*
3.	$q \rightarrow r$	*Premise*
4.	p	Disjunctive syllogism from (1) and (2)

Step (4) means that "I am clever".

b) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephants running down the road."

(f) Let us assume:

p ="I am dreaming"

q ="I am hallucinating"

r ="I see elephants running down the road"

We can then rewrite the given statements using the above interpretations.

	Step	Reason
1.	$p \vee q$	*Premise*
2.	$\neg p$	*Premise*
3.	$q \to r$	*Premise*
4.	q	Disjunctive syllogism from (1) and (2)
5.	r	Modus ponens from (3) and (4)

Step (4) means that "I am hallucinating".

Step (5) means that "I see elephants running down the road".

Question 3 CLO-3 Marks 8

1) **Find f ◦ g, where f (x) = $2x^2 + 1$ and g(x) = 3x + 2, are functions from R to R. (2)**

Sol: fog = ? $f(x) = 2x^2+1$ $g(x) = 3x+2$

since f and g are both functions from R to R, fog is also function from R to R.

using definition of composition:-

$(fog)(x) = f(g(x)) = f(3x+2) = 2(3x+2)^2 + 1$

$= 2(9x^2 + 12x + 4) + 1$

$= 18x^2 + 24x + 8 + 1$

$= 18x^2 + 24x + 9.$

2) **Determine whether each of the following functions is a bijection from R to R.**
$f(x) = x^5+1$. (1)

Part d -

$f(x) = x^5 + 1$ is a bijection.

It is a strictly increasing function. Use exercise 26

3) **Write the set builder notation of set A= {0, 1, 4, 9, 16, 25, 36, 49, 64, 81} (1)**

$\{x \mid x = n^2 \text{ where } n \in W \quad 0 \leq n \leq 9\}$

4) **Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}.**
Find (A -B) ∪ (A∩ C)

3(4) $A - B = \{8, 10\}$.
$A \cap C = \{4, 6, 8, 10\}$.
$(A - B) \cup (A \cap C) = \{4, 6, 8, 10\}$

(1)

5) **Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order).**
R={(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)} (1)

The a_{ij}th element of the matrix is 1 when (i, j) is an element in the relation.
The a_{ij}th element of the matrix is 0 when (i, j) is an element in the relation.

$\{(1,2)(1,3)(1,4)(2,1)(2,3)(2,4)(3,1)(3,2)(3,4)(4,1)(4,2)(4,3)\}$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

6) **Determine whether the relation represented by the matrice in question 5 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive. (2)**
Reflexive = no
Irreflexive = yes
Symmetric = yes
Asymmetric = no
Transitive = yes